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View Factor from Conical Surface by Contour Integration

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Introduction

The prediction of radiative heat transfer from the engine exhaust plume to the stage is necessary to evaluate thermal protection requirements. Therefore, the radiation view factor between the plume and the stage must be evaluated to determine the radiant heat rates. The normal approach to this problem is to assume that the exhaust plume behaves as a radiating cone; thus the problem is reduced to determining the view factor between a conical surface and a differential area on the stage. Using this approach, Morizumi[1] and Bobco[2] attempted to evaluate the view factor from a differential area to a cone. Morizumi used the Nusselt double-projection method. His analysis has the disadvantage of determining the view factor to a conical surface that is truncated by the differential-area line of sight instead of by a plane perpendicular to the cone axis. Bobco attempted to develop an expression for the view factor by integrating the general equation for view factors;

$$F_{dA_1-A_2} = \int\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2 dA_2} \qquad (1)$$

where θ_1, θ_2, and r are indicated in Fig. 1. He successfully integrated Eq. (1) once but was then forced to integrate the resulting expression numerically.

Analysis

An analysis is developed here that will yield the view factor from a differential area to a cone. By applying Stokes theorem[3] to Eq. (1) the view-factor equation is reduced to contour integrals. For the configuration of interest (Fig. 1) the differential-area normal is parallel to the cone axis. The view factor for this configuration can therefore be represented

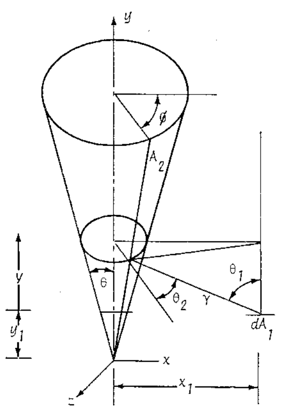

Fig. 1 View factor from differential area to cone.

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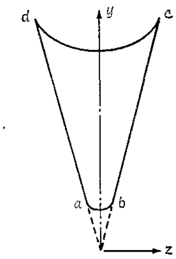

Fig. 2 Contour lines.

as a single contour integral

$$F_{dA_1-A_2} = \oint_{c_2} \frac{(x - x_1)dz - zdx}{2\pi r^2} \qquad (2)$$

This contour integral is integrated around the path that is the limit of sight on the cone from the differential area. The contour path is shown in Fig. 2.

To integrate Eq. (2), x and z are written in polar form using the cone radius R and the angle ϕ measured from the x axis in the $x - z$ plane. The cone radius is

$$R = y \tan\theta \qquad (3)$$

where θ is the half angle of the cone. Employing symmetry and expanding Eq. (2) into regular integrals yields

$$F_{dA_1-A_2} = \int_0^{-\phi_0} \frac{(y_1{}^2 \tan^2\theta - x_1 y_1 \tan\theta \cos\phi)d\phi}{\pi r^2} + \int_{y_1}^{\bar{y}} \frac{x_1 \tan\theta \sin\phi_0 dy}{\pi r^2} + \int_{-\phi_0}^{0} \frac{(\bar{y}^2 \tan^2\theta - x_1\bar{y} \tan\theta \cos\phi)d\phi}{\pi r^2} \qquad (4)$$

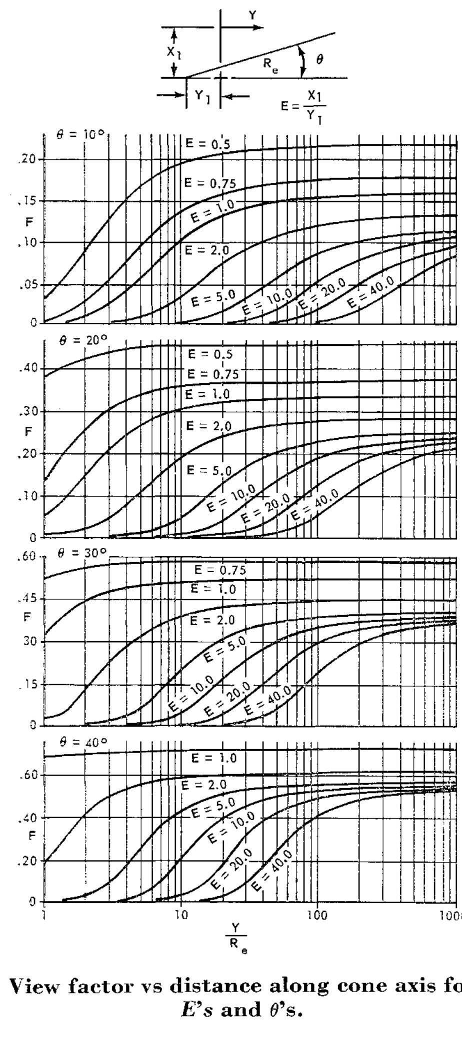

Fig. 3 View factor vs distance along cone axis for various E's and θ's.

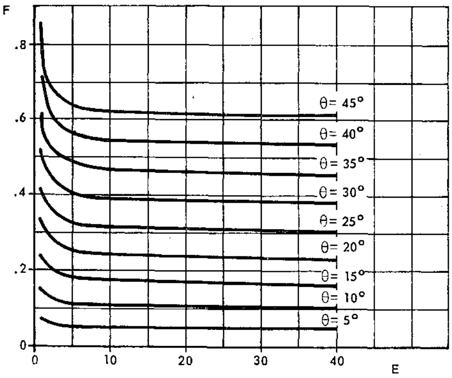

Fig. 4 Limiting view factor.

From Fig. 1 the equation for r^2 is

$$r^2 = y_1^2[(y/y_1)^2 \sec^2\theta - 2(y/y_1)\{1 + (x_1/y_1)\tan\theta\cos\phi\} + (x_1/y_1)^2 + 1] \quad (5)$$

Equation (5) is substituted into Eq. (4) and the resulting expression is integrated. The angle at the limit of sight on the conical surface is

$$\phi_0 = \cos^{-1}[(y_1/x_1)\tan\theta] \quad (6)$$

Substituting Eq. (6) for ϕ_0 and putting the solution in nondimensional form yields

$$F_{dA_1-A_2} = \frac{\sin\theta}{\pi}\tan^{-1}\left[\frac{\sec\theta}{(E^2 - \tan^2\theta)^{1/2}}(\bar{\eta} - 1)\right] + \frac{1}{\pi}\tan^{-1}\left[\frac{E + \tan\theta}{E - \tan\theta}\right]^{1/2} - \frac{(\bar{\eta} - 1)^2 + E^2 - \bar{\eta}^2\tan^2\theta}{[(\bar{\eta}\tan\theta + E)^2 + (\bar{\eta} - 1)^2]^{1/2}[(\bar{\eta}\tan\theta - E)^2 + (\bar{\eta} - 1)^2]^{1/2}} \times \tan^{-1}\left\{\left[\frac{(\bar{\eta}\tan\theta + E)^2 + (\bar{\eta} - 1)^2}{(\bar{\eta}\tan\theta - E)^2 + (\bar{\eta} - 1)^2}\right]^{1/2}\left(\frac{E - \tan\theta}{E + \tan\theta}\right)^{1/2}\right\} \quad (7)$$

where $\eta = y/y_1$ and $E = x_1/y_1$. Equation (7) is the expression for the view factor from the differential area to the conical surface. The only restrictions on this equation are that $E \geq \tan\theta$ and $y > 0$.

On the conical boundary ($E = \tan\theta$), Eq. (7) reduces to

$$F_{dA_1-A_2} = \sin\theta/2 + \tfrac{1}{2} \quad (8)$$

In some applications, the limiting value of the view factor as the cone length approaches infinity is of interest. From Eq. (7)

$$\lim_{\bar{\eta}\to\infty} F_{dA_1-A_2} = \frac{\sin\theta}{2} + \frac{1}{\pi}\tan^{-1}\left[\frac{E + \tan\theta}{E - \tan\theta}\right]^{1/2} - \frac{1}{\pi}\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\tan^{-1}\left[\frac{E - \tan\theta}{E + \tan\theta}\right]^{1/2} \quad (9)$$

Figure 3 shows graphs of the view factor vs the distance down the cone axis for cone half-angles of 10°, 20°, 30°, and 40°. This distance is measured from the differential area (at $Y = 0$) and nondimensionalized with respect to the cone radius at the differential-area y position. Y/R_e is a more convenient parameter than η. The relation between the two nondimensional distances is

$$Y/R_e = (\eta - 1)/\tan\theta \quad (10)$$

Figure 4 is a graph of the limiting view factor (Eq. 10) vs the differential-area position factor E for various cone half-angles. This graph shows that for $E > 10$ increasing E has little effect on the limiting view factor.

Conclusions

The analysis presents a more useable result than the Bobco and Morizumi approaches because it yields an algebraic equation for the view factor. This enables rapid calculation of view factors with the slide rule or calculation machine. All of the data presented in Figs. 3 and 4 were obtained from a digital computer program that required less than 2 min execution time.

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Dynamic Plastic Response of Finite Bars

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Introduction

THIS Note examines the dynamic response of a fixed-free, finite bar subjected to an axially applied impact load where the load is sufficiently high to result in plastic strains (Fig. 1). The material of the bar is represented by a bilinear static stress-strain diagram where the elastic and plastic moduli are denoted by E_0 and E_1, respectively. By varying the slope of the plastic region of this curve (E_1/E_0), it is possible to explore the effects of strain hardening on this problem, including the limiting case of elastic wave propagation in which E_1/E_0 is equal to one.

Analysis

In the problem considered, the time history of applied normal stress at the end $x = 0$ is given by

$$\sigma(0,t) = 0,\ t \leq 0 \quad (1)$$

$$\sigma(0,t) = \mathrm{P},\ t > 0 \quad (2)$$

where $P > \sigma_0$ (yield stress). In this example the applied stress was selected such that $P = 4\sigma_0$. The boundary condition at the fixed end of the bar is that the particle velocity is zero at $x = L$ for all time, or

$$v(L,t) = 0 \quad (3)$$

For one-dimensional plastic waves the rate-dependent theory of Malvern[1,2] results in the following set of quasilinear partial differential equations:

$$\partial\sigma/\partial x = \rho\partial v/\partial t \quad (4)$$

$$\partial\epsilon/\partial t = \partial v/\partial x \quad (5)$$

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